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Privatization and Technology Licensing in Mixed Oligopoly

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Abstract

This paper examines the optimal privatization decision in a mixed oligopoly where technology transfer takes place from a cost-efficient foreign private firm to a cost-inefficient domestic public firm via two-part tariff licensing. Partial privatization is possible under Bertrand competition. Partial or full privatization is found optimal under Cournot competition based on the cost differential even with constant returns to scale technology, product differentiation and two-part tariff licensing. This paper has also investigated the relation between the optimal degree of privatization and the degree of product substitutability for a given cost differential in case of both competitions.

Keywords: *Privatization, Mixed Oligopoly, Technology Licensing, Competition.*
JEL classification: *D43, D45, D86, L33*

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1 Introduction

Privatization often sparks strong debates in news, social media, public spheres as well as academia in developing countries. Being continuously promoted by the Government of India since the economic liberalization in 1991, the process of privatization of public sector enterprises took pace in recent years after the finance minister Nirmala Sitaraman announcing privatization policy for all government establishments including central public sector enterprises for strategic and non-strategic sectors.[1] Central public sector enterprises like Air India and Central Electronics Ltd were privatized in 2021.

Advocates of privatization majorly point towards the relative inefficiency of public firms compared to the private firms and suggest privatization as an effective tool of reducing inefficiency via cost reduction (Choi 2019). The oppositions of the policy indicate the detrimental effect on social welfare due to the shrink in the welfare-oriented objective of the public firms (Matsumura 1998). In the backdrop of this trade-off the optimal level of privatization is always an intriguing question in the economic literature.

One possible way-out to achieve cost-efficiency for the public firms is technology licensing form a cost-efficient foreign private firm to the cost-inefficient domestic firm. It is widely observed in developing countries with mixed oligopolies. For example, the German BMW Motor Corporation licensed its engine technology to Chinese state-owned Dongfeng Motor Corporation for the production of Fengxing T5 SUV in 2018. However, the issues of technology licensing and privatization decisions in case of mixed oligopolies are not often discussed connectedly, even when the real world examples indicate strong interrelation. In this paper, we have tried to enrich the literature stressing on this interrelation.

There is a growing literature on licensing in mixed oligopolies. Chen et al.(2014) examined the optimal licensing when a private firm is licensing to a public firm and a private rival firm in a mixed oligopoly. Kim et al.(2018) constructed a model where a foreign innovator is licensing eco-technology in a polluting mixed duopoly under emission tax and cost asymmetry between the public and the private firm. Yan and Yang(2018) considered optimal licensing schemes in the presence of uncertain R & D outcomes and technology spillover when a mixed owner is the licenser. Chen et al.(2021) examined a partially privatized firm in a mixed duopoly under R & D competition with discriminatory output subsidies. Nie and Yang(2020) examined several cost-reducing innovation strategies under a mixed duopoly and concluded that the Government should encourage partial nationalization. [2] However, all of the above studies emphasize on the choice of licensing contract over the privatization decision.

There exists a rich literature on the optimal privatization decision in the mixed oligopolies with product differentiation. Fujiwara(2007) showed that in case of no efficiency gap between the public and the private firm in a mixed oligopoly with product differentiation, the public firm will be partially privatized when the firms engage in quantity competition. Bárcena-Ruiz and Garzón (2020) examined the privatization decision in an international mixed market where more than one government is taking the privatization decision and ended with partial privatization in case of Cournot competition when

[1]"Public sector enterprises in non-strategic sectors to be privatized", Livemint, 17 May 2020, retrieved 18 May 2020.

[2]All the above examples are of private licensing, where a private firm licensed its technology to a public or semi-public firm. However, there also exist several studies those examined the case of public licensing, i.e., licensing by a public firm to a private firm. See Ye(2012), Gelves and Heywood(2016), Heywood et al.(2019) for details. In our paper, we have considered only private licensing.

the goods are close substitutes. Ishibashi and Kaneko(2008) considered a price setting game with quality competition in a mixed duopoly. They found that partial privatization is never optimal if quality is exogenous. But if the qualities are chosen before prices, then partial privatization is optimal. In a similar setup of Fujiwara(2007), Ohnishi(2010) showed that the public firm will not be privatized at all under Bertrand competition. Liu et al.(2019) found that for a differentiated vertically related oligopoly with a upstream semi-public firm, no privatization is optimal for relatively high product differentiation and full privatization is optimal for relatively low product differentiation under price competition. In summary, partial privatization is found to be optimal in case of quantity competition in a differentiated mixed oligopoly, while partial privatization is generally not optimal for price competition in a differentiated mixed oligopoly. But these studies focused on the optimal privatization decision, and did not consider technology licensing.

Only a few studies capture the interaction between the optimal privatization decision and technology licensing in a mixed oligopoly. Mukherjee and Sinha (2014) constructed a model of mixed duopoly with a technologically superior private firm and a welfare-maximizing public firm and found that under technology licensing there is no need for privatization. Niu(2015) concluded that when a cost-reducing technology of a foreign innovator is faced with a domestic public monopoly, the public monopoly will be charged higher compared to a domestic private firm and therefore, the Government's optimal response will be to partially privatize the public monopoly. Wang and Zeng(2019) studied the effect of licensing decision of an efficient private firm to either a foreign private firm or a public firm on the level of privatization. They concluded that licensing to the foreign private firm encourages the privatization while licensing to the public firm discourages privatization. The model deals with exogenous licensing regimes with endogenous privatization decision. Haruguchi and Matsumura(2020) examined a mixed triopoly with a public domestic firm, a private domestic firm and a private foreign firm. They showed that the foreign firm voluntary transfers its technology in case of endogenous privatization policy. In this case, the firm's choice of technology transfer is followed by an endogenous privatization policy. Wang et al.(2020) examined the relationship between the technology licensing and privatization for a mixed duopoly with a profit maximizing foreign firm and a public firm. The cost efficient firm licenses its superior technology to the rival firm in exchange of a fixed fee. They concluded that in case of a foreign private firm, private licensing facilitates privatization. Shastri and Sinha(2024) studied the relationship in presence of barriers to technology transfer, i.e., the cost-inefficient public firm's budget constraint to finance the technology transfer. The outcome will be full or partial privatization based on initial cost difference. However, all these papers have considered only Cournot competition and ignored the Bertrand competition in the final stage of the game.

In our paper, we have studied the optimal privatization decision in a mixed oligopoly where a foreign private firm transfers its superior technology to a cost-inefficient publicly regulated firm via two-part tariff mechanism. We have constructed a mixed duopoly model with differentiated goods where the technology transfer eliminates the cost differential between the domestic public and foreign private firm. We have considered the following game: at stage-I, the government decides the optimal degree of privatization based on welfare maximization of the economy; at stage-II, the foreign private firm determines the two-part tariff based on its profit maximization and the surplus in the objective function of the public firm due to technology transfer; and at stage-III, the both firms simultaneously and independently decides its quantity(price) decisions according to Cournot(Bertrand) competition. We solved the game via backward induction and found the Sub-game Perfect Nash equilibria for the game. We have derived the optimal privatization decision with and without licensing agreement under

Bertrand and Cournot competition and have compared them. We have also tried to investigate how the factors like cost differential and degree of product substitutability affects the optimal privatization decision.

Our paper is closely related to Wang et al.(2020) but differs in the following ways. Firstly, they considered homogeneous products in their model, where we have allowed for differentiated products. Secondly, they considered decreasing returns to scale technologies while we have considered constant returns to scale technologies of production. Thirdly, they assumed technology licensing by the means of a fixed fee. In our paper, we have assumed technology licensing via two-part tariff mechanism, i.e., a combination of a fixed fee and a per unit royalty fee. Lastly, while they considered that the firms in that mixed oligopoly engage only in quantity competition, we have considered both the quantity and price competition and tried to provide a comparison between them.

Our paper contributes to the existing literature in several ways as well as it has policy implications. Firstly, the earlier literature have shown that the optimal privatization decision of the Government for price competition in a mixed duopoly is either full privatization or no privatization at all (Ohnishi, 2010). Under our setup, we came with partial privatization for that case under certain conditions. Secondly, we have checked the robustness of the existing results of optimal privatization under quantity competition. The results derived by Wang et al.(2020) for quantity competition in a mixed duopoly with a public firm and a foreign private firm with technology licensing even if we allow for product differentiation, constant returns to scale technology and technology transfer via two-part tariff mechanism. Thirdly, we found that the technology adoption condition of the licensing contract under the optimal degree of privatization not only depends on the supply side parameter like cost differential as suggested by the existing literature (Chen et al.,2014) but on the demand side parameter like the degree of product substitutability also. Fourthly, we provide a Cournot-Bertrand comparison of the optimal level of privatization in case of a mixed duopoly where a cost-efficient foreign private firm is transferring its superior technology via two-part tariff licensing to a cost-inefficient public firm. In presence of technology licensing, Cournot level of optimal privatization is expected to be greater than the Bertrand level of optimal privatization in a differentiated mixed duopoly. However, we have found that while this ranking holds for high degree of product differentiation and cost differential, the ranking got reverse for both the degree of product substitutability and cost differential being low enough. Finally, earlier studies found an inverted-U shaped relationship between the optimal level of privatization and the degree of product substitutability at given level of cost differential in case of Cournot competition (Fujiwara, 2007). Here, in our model, the inverted-U relationship is found for the cost differential given at low level, but eventually it got U-shaped for sufficiently high level of given cost differential. Additionally, we found that in case of Bertrand competition the relationship between the two is U-shaped if the given level of cost differential is low but is decreasing if the given level of cost differential is sufficiently high. [3]

Rest of the paper is organized as follows. In section 2, the basic model is described. We derive the optimal privatization decision under Bertrand competition for both licensing and no licensing case in the section 3, examining the same under Cournot competition in section 4. Section 5 provides a deeper look on the technology adoption condition that must hold to realize the licensing contract. In section 6, we compare the optimal level of privatization for Cournot and Bertrand competition. Section 7 deals with the relationship between the degree of product substitutability and the optimal level of privatization under both the price and quantity competition. Lastly, in section 8 we conclude.

[3] Dadpay et al.(2022) provides the comparison between Cournot and Stackleberg outcomes in context of licensing behavior in an international mixed duopoly with a mixed ownership public firm and a foreign private firm.

2 The model

We consider a economy with two sectors, a competitive sector producing a nuemeraire commodity (money) and an imperfectly competitive sector producing differentiated commodities. The imperfectly competitive sector comprises of two firms: a publicly regulated firm (say, Firm 1) and a foreign private firm (Firm 2).

2.1 Demand side

The utility of a representative consumer can be described as follows:

$$U(q_1, q_2, m) = m + a(q_1 + q_2) - \frac{1}{2}[q_1^2 + q_2^2 + 2\beta q_1 q_2]; a > 0, \beta \in (0, 1) \tag{1}$$

where m is the consumption of the nuemeraire commodity, a is the taste parameter and β is the degree of product substitution.The quantity produced by the Firms 1 and 2 are given by q_1 and q_2 respectively.

Given the quasi-linear utility function of the representative consumer, the consumer will maximize its utility subject to its budget constraint revealing the following inverse demand function of firm i

$$p_i(q_1, q_2) = \frac{\partial U(q_1, q_2)}{\partial q_i} = a - q_i - \beta q_j \forall i, j = 1, 2 \& i \neq j \tag{2}$$

This inverse demand function is invertible given $\beta \in (0, 1)$ and after solving for q_i we derive the direct demand function of firm i as:

$$D_i(p_1, p_2) = \frac{a}{1+\beta} - \frac{p_i - \beta p_j}{1 - \beta^2} \forall i, j = 1, 2 \& i \neq j \tag{3}$$

Without any loss of generality and for the sake of computational ease, we assume the taste parameter, $a = 1$.

Therefore, the consumer surplus can be given as

$$CS(q_1, q_2) = U(q_1, q_2) - p_1(q_1, q_2)q_1 - p_2(q_1, q_2)q_2 \tag{4}$$

using the price-quantity duality, we can alternatively represent the consumer surplus in terms of prices:

$$CS^B(p_1, p_2) = U(D_1(p_1, p_2), D_2(p_1, p_2)) - p_1 D_1(p_1, p_2) - p_2 D_2(p_1, p_2) \tag{5}$$

2.2 Supply side

We consider a duopoly in the imperfectly competitive sector with differentiated commodities. We assume the foreign private firm is more cost-efficient compared to the domestic public firm. We also assume constant marginal cost of production for both the firms. The cost structure for the firm i is as follows:

$$C_i = c_i q_i \tag{6}$$

Without any loss of generality, we are setting $c_1 = c(\in (0, 1))$ and $c_2 = 0$.

Thus, the profit functions of the firms under no licensing contract is respectively as follows:

$$\Pi_1^N = (p_1 - c)q_1$$

$$\Pi_2^N = p_2 q_2$$

We assume the more cost-efficient foreign private firm gives the license to use its superior technology to the less cost-efficient public firm through two-part tariff scheme. That is, the private firm charges a fixed fee, f and a per-unit royalty charge, ω. After licensing, the publicly regulated firm becomes efficient. Under this two-part tariff licensing scheme, the profit functions of the respective firms becomes:

$$\Pi_1^L = (p_1 - \omega)q_1 - f$$

$$\Pi_2^L = p_2 q_2 + \omega q_1 + f$$

The objective of the foreign private firm is the profit maximization. The public firm's objective function is the weighted average of its own profit and the total welfare [Matsumura, 1998]. It is represented as follows:

$$V = \theta \Pi_1 + (1 - \theta)W$$

2.3 Social welfare

The total surplus of the economy is given by the sum of the consumers surplus and the profit of the domestic public firm.

$$W = CS + \Pi_1 \tag{7}$$

2.4 Game Structure

The structure of the mixed oligopoly licensing game is as follows:

- **Stage-I** The Government decides the level of privatization ($\theta \in [0, 1]$) maximizing social welfare. If $\theta = 0$, no privatization is allowed in the public firm and if $\theta = 1$, the public firm is fully privatized.
- **Stage-II** The cost-efficient foreign private firm will decide the two-part tariff, i.e., how much f and ω to impose upon the public firm. The per unit royalty fee will be decided based on the maximization of profit of the private firm and the fixed fee will be equal to the surplus in the objective function of the public firm under licensing contract compared to no licensing situation.
- **Stage-III** Finally, the both firms simultaneously and independently decide its quantity(price) decisions according to Cournot(Bertrand) competition.

We solve the game using the backward induction method.

3 Privatization decision under Bertrand competition

3.1 Bertrand competition under no licensing

Under no licensing Bertrand competition the three stage game is reduced to a two stage game- (I) the Government decides the extent of privatization and (II) the both firms decide how much price to set simultaneously and independently taking quantity produced as given according to Bertrand competition.[4]

The profit functions of firm 1 and 2 are respectively as follows:

$$\Pi_1^{NB}(p_1, p_2) = (p_1 - c)D(p_1, p_2) \tag{8}$$

$$\Pi_2^{NB}(p_1, p_2) = p_2 D_2(q_1, q_2) \tag{9}$$

The social welfare in this case is

$$W(p_1, p_2) = CS(p_1, p_2) + \Pi_1^{NB}(p_1, p_2) \tag{10}$$

The objective function of the public firm(firm 1) is

$$V^{NB}(p_1, p_2) = \theta\Pi_1^{NB}(p_1, p_2) + (1-\theta)W^{NB}(p_1, p_2) \tag{11}$$

Firm 1 will maximize its objective function to decide how much price to charge. To maximize, after differentiating with respect to p_1 and equating to 0, we get the following first order condition:

$$\frac{\partial V^{NB}(p_1, p_2)}{\partial p_1} = \theta\frac{\partial \Pi_1^{NB}(p_1, p_2)}{\partial p_1} + (1-\theta)\frac{\partial W^{NB}(p_1, p_2)}{\partial p_1} = 0 \tag{12}$$

Firm 2 will decide its production maximizing its Profit. We differentiate $\Pi_2^{NB}(p_1, p_2)$ with respect to p_2 and equate it with 0 to get the following first order condition:

$$\frac{\partial \Pi_2^{NB}(p_1, p_2)}{\partial p_2} = p_2\frac{\partial D_2(p_1, p_2)}{\partial p_2} + D_2(p_1, p_2) = 0 \tag{13}$$

solving (12) and (13) for p_1 and p_2, we get

$$p_1^{\hat{N}B} = \frac{(\beta^2 + \beta - 2)\theta - 2c}{\theta\beta^2 - 2\theta - 2} \tag{14}$$

$$p_2^{\hat{N}B} = \frac{\theta\beta^2 - c\beta - \theta + \beta - 1}{\theta\beta^2 - 2\theta - 2} \tag{15}$$

Using (14) and (15) in equations (3), we get

$$q_1^{\hat{N}B} = \frac{(c-1)\beta^2 - 2c - \beta + 2}{(\beta^2 - 1)(\theta\beta^2 - 2\theta - 2)} \tag{16}$$

[4]We will use 'N' in the superscript to denote no licensing situation and 'L' to denote licensing situation. Similarly, we write 'B' in the superscript to mean Bertrand competition and 'C' to mean Cournot competition.

$$q_2^{\hat{NB}} = \frac{-\theta\beta^2 + c\beta + \theta - \beta + 1}{(\beta^2 - 1)(\theta\beta^2 - 2\theta - 2)} \tag{17}$$

Using (14) and (15) in equations (8), (9), (5), (10) and (11) we get the following:

$$\Pi_1^{\hat{NB}} = \Pi_1^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}}) \tag{18}$$

$$\Pi_2^{\hat{NB}} = \Pi_2^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}}) \tag{19}$$

$$C\hat{S}^{NB} = CS(p_1^{\hat{NB}}, p_2^{\hat{NB}}) \tag{20}$$

$$W^{\hat{NB}} = W^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}}) \tag{21}$$

$$V^{\hat{NB}} = V^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}}) \tag{22}$$

Now, the Government will determine the level of privatization maximizing the social welfare. Differentiating $W^{\hat{NB}}$ with respect to θ, we get the following first order condition:

$$\frac{\partial W^{\hat{NB}}}{\partial\theta} = \frac{\partial W^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}})}{\partial p_1^{\hat{NB}}}\frac{\partial p_1^{\hat{NB}}}{\partial\theta} + \frac{W^{NB}(p_1^{\hat{NB}}, p_2^{\hat{NB}})}{\partial p_2^{\hat{NB}}}\frac{\partial p_2^{\hat{NB}}}{\partial\theta} = 0 \tag{23}$$

Solving (23), we get $\theta^{NB}(c,\beta)$ from which we obtain optimal level of privatization $\theta^{*NB}(c,\beta)$ as follows

$$\theta^{*NB}(c,\beta) = \begin{cases} 0 & \textbf{if} \quad \theta^{NB}(c,\beta) < 0 \\ \theta^{NB}(c,\beta) & \textbf{if} \quad 0 < \theta^{NB}(c,\beta) < 1 \\ 1 & \textbf{if} \quad \theta^{NB}(c,\beta) > 1 \end{cases} \tag{24}$$

In terms of cost differential, it becomes

$$\theta^{*NB}(c,\beta) = \begin{cases} 0 & \textbf{if} \quad c < \bar{c} \\ \{0,1\} & \textbf{if} \quad c = \bar{c} \\ 1 & \textbf{if} \quad c > \bar{c} \end{cases} \tag{25}$$

where $\bar{c} = \frac{\beta^4 - 4\beta^2 - \beta + 4}{(\beta^2 - 2)^2}$

3.2 Bertrand competition under licensing

For Bertrand competition under licensing, the profit function of firm 1 and firm 2 becomes respectively

$$\Pi_1^{LB}(p_1, p_2) = (p_1 - \omega)D_1(p_1, p_2) - f \tag{26}$$

$$\Pi_2^{LB}(p_1, p_2) = p_2 D_2(p_1, p_2) + \omega D_1(p_1, p_2) + f \tag{27}$$

The social welfare function becomes

$$W^{LB}(p_1, p_2) = CS(p_1, p_2) + \Pi_1^{LB}(p_1, p_2) \tag{28}$$

The objective function of firm 1 in this case is

$$V^{LB}(p_1, p_2) = \theta\Pi_1^{LB}(p_1, p_2) + (1-\theta)W^{LB}(p_1, p_2) \tag{29}$$

At stage III, the firms will decide prices taking quantity produced given according to Bertrand setup. To maximize firm 1's objective function, differentiating $V^{LB}(p_1, p_2)$ with respect to p_1 and equating it with 0, we get the following first order condition:

$$\frac{\partial V^{LB}(p_1, p_2)}{\partial p_1} = \theta\frac{\partial \Pi_1^{LB}(p_1, p_2)}{\partial p_1} + (1-\theta)\frac{\partial W^{LB}(p_1, p_2)}{\partial p_1} = 0 \tag{30}$$

For maximization of firm 2's profit, we differentiate $\Pi_2^{LB}(p_1, p_2)$ with respect to p_2 and equate it with 0 to obtain the following first order condition:

$$\frac{\partial \Pi_2^{LB}(p_1, p_2)}{\partial p_2} = p_2\frac{\partial D_2(p_1, p_2)}{\partial p_2} + D_2(p_1, p_2) + \omega\frac{\partial D_1(p_1, p_2)}{\partial p_2} = 0 \tag{31}$$

Using (30) and (31) we solve for p_1 and p_2, we get

$$p_1^{\hat{LB}} = \frac{((1-\omega)\beta^2 + \beta - 2)\theta - 2\omega}{\theta\beta^2 - 2\theta - 2} \tag{32}$$

$$p_2^{\hat{LB}} = \frac{\theta\beta^2 - \theta\beta\omega - 2\beta\omega - \theta + \beta - 1}{\theta\beta^2 - 2\theta - 2} \tag{33}$$

Putting (32) and (33) in (3), we get

$$q_1^{\hat{LB}} = \frac{2\beta^2\omega - \beta^2 - \beta - 2\omega + 2}{(\beta^2-1)(\theta\beta^2 - 2\theta - 2)} \tag{34}$$

$$q_2^{\hat{LB}} = \frac{(\beta-1)((\beta+1)(\beta\omega-1)\theta) - 1}{(\beta^4 - 3\beta^2 + 2)\theta - 2\beta^2 + 2} \tag{35}$$

Also using (32) and (33) in (26), (27), (5), (28) and (29), we get

$$\Pi_1^{\hat{LB}} = \Pi_1^{LB}(p_1^{\hat{LB}}, p_2^{\hat{LB}}) \tag{36}$$

$$\Pi_2^{\hat{LB}} = \Pi_2^{LB}(q_1^{\hat{LB}}, q_2^{\hat{LB}}) \tag{37}$$

$$C\hat{S}^{LB} = CS(p_1^{\hat{LB}}, p_2^{\hat{LB}}) \tag{38}$$

$$W^{\hat{LB}} = W^{LB}(p_1^{\hat{LB}}, p_2^{\hat{LB}}) \tag{39}$$

$$V^{\hat{LB}} = V^{LB}(p_1^{\hat{LB}}, p_2^{\hat{LB}}) \tag{40}$$

In stage II, the private firm(firm 2) will decide the two-part tariff. The fixed fee (f) will be calculated based on the surplus in the objective function the public firm(firm 1) enjoys under licensing contract than the no licensing scenario. Using (22) and (40) we calculate the fixed fee charged as:

$$f^{LB} = V^{\hat{LB}} - V^{\hat{NB}} = \frac{1}{2(\beta^2-1)(\theta\beta^2 - 2\theta - 2)^2}\sum_{i=0}^{4}\Psi_i(\omega, c, \beta)\theta^i \tag{41}$$

where $\Psi_3(\omega, c, \beta) = -\beta^4\omega^2 + 2\beta^3\omega + \beta^2\omega^2 2\beta\omega$, $\Psi_2(\omega, c, \beta) = (-3\omega^2 + 2\omega)\beta^4 - 2(-2\omega + c)\beta^3 + (3\omega^2 2\omega)\beta^2 + 2(-2\omega + c)\beta$, $\Psi_1(\omega, c, \beta) = 2(c^2 2\omega^2 - 2c + 3\omega)\beta^4 - 2(c - \omega)\beta^3 + (-5c^2 + 8\omega^2 + 10c14\omega)\beta^2 + 2(-\omega + 2c)\beta + 4(c - 2 + \omega)(c - \omega)$ and $\Psi_0(\omega, c, \beta) = -3c^2 + 4\omega^2 + 6c8\omega)\beta^2 + 2c\beta + 4(c - 2 + \omega)(c - \omega$.

Putting (41) in equation (37), we get

$$\tilde{\Pi_2^{LB}} = \{\hat{\Pi_2^{LB}} | f = f^{LB}\} \tag{42}$$

Now, in order to maximize firm 2's profit, we differentiate $\tilde{\Pi_2^{LB}}$ with respect to ω and equate it with 0 to derive the first order condition. Solving the first order condition, we find the optimal ω as follows:

$$\omega^{*LB} = \frac{\theta\beta(\theta^2 + (\beta^2 + \beta + 2)\theta + 3\beta + 1)}{(\theta + 1)(\theta^2\beta^2 + 4\theta\beta^2 + 4)} \tag{43}$$

Putting (43) in (41), we get the final value of optimal fixed fee under licensing as

$$f^{*LB} = \{f^{LB} | \omega = \omega^{*LB}\} \tag{44}$$

Technology Adaption Condition (TAC): any rational firm will choose a contract only if its cost will reduce when under licensing contract compared to the situation with no-licensing. Hence, we impose the rationality assumption for firm 1, i.e., the cost-inefficient public firm will go for the contract only when the tariff rate under licensing is less than its original marginal cost under no-licensing.

$$\omega^{*LB} < c \tag{45}$$

Replacing (43) and (44) in (32), (33), (34), (35) we get,

$$\bar{p_1}^{LB} = \frac{(\theta^2\beta^2 + (-\beta^3 + 4\beta^2 + \beta)\theta + \beta^2 - \beta + 4)\theta}{(\theta + 1)(\theta^2\beta^2 + 4\theta\beta^2 + 4)} \tag{46}$$

$$\bar{p_2}^{LB} = \frac{\theta^3\beta^2 + 4\theta^2\beta^2 + 2(\beta^2 + 1)\theta - 2(\beta - 1)}{(\theta + 1)(\theta^2\beta^2 + 4\theta\beta^2 + 4)} \tag{47}$$

$$\bar{q_1}^{LB} = \frac{2\theta\beta^2 - (\theta^2 + \theta - 2)\beta + 4}{(\beta + 1)(\theta + 1)(\theta^2\beta^2 + 4\theta\beta^2 + 4)} \tag{48}$$

$$\bar{q_2}^{LB} = \frac{\beta^2(\beta + 2)\theta^2 + (3\beta^2 + 2\beta + 2)\theta + 2}{(\beta + 1)(\theta + 1)(\theta^2\beta^2 + 4\theta\beta^2 + 4)} \tag{49}$$

Putting (46) and (47) in (36), (37), (38), (39) we get,

$$\bar{\Pi_1}^{LB} = \hat{\Pi_1^{LB}}(\bar{p_1}^{LB}, \bar{p_2}^{LB}) \tag{50}$$

$$\bar{\Pi_2}^{LB} = \hat{\Pi_2^{LB}}(\bar{p_1}^{LB}, \bar{p_2}^{LB}) \tag{51}$$

$$\bar{CS}^{LB} = C\hat{S}^{LB}(\bar{p_1}^{LB}, \bar{p_2}^{LB}) \tag{52}$$

$$\bar{W}^{LB} = \hat{W^{LB}}(\bar{p_1}^{LB}, \bar{p_2}^{LB}) \tag{53}$$

Finally, in stage I, the government will decide the extent of privatization, θ^{LB}, maximizing the social welfare. Using (53), differentiating $\bar{W}^{LB}$ with respect to θ and equating it with 0, we get the following first order condition,

$$\frac{\partial \bar{W}^{LB}(\bar{p_1}^{LB}, \bar{p_2}^{LB})}{\partial \theta} = \frac{\partial \bar{W}^{LB}(\bar{p_1}^{LB}, \bar{p_2}^{LB})}{\partial \bar{p_1}^{LB}} \frac{\partial \bar{p_1}^{LB}}{\partial \theta} + \frac{\partial \bar{W}^{LB}(\bar{p_1}^{LB}, \bar{p_2}^{LB})}{\partial \bar{p_2}^{LB}} \frac{\partial \bar{p_2}^{LB}}{\partial \theta} = 0 \tag{54}$$

solving (54), we get $\theta^{LB}(c, \beta)$ from which we obtain the optimal level of privatization under licensing in Bertrand competition, $\theta^{*LB}(c, \beta)$.[5]

$$\theta^{*NB}(c, \beta) = min[0, \theta^{LB}(c, \beta)] \tag{55}$$

Now, we can summarize our findings under Bertrand competition in the following proposition.

Proposition 1. *Under Bertrand competition, the following holds:*

(i) The cost inefficient public firm will not be privatized at all or fully privatized based on the original marginal cost of the public firm with no licensing contract. No partial privatization is optimal.

$$\theta^{*NB}(c, \beta) = \begin{cases} 0 & \textbf{\textit{if}} \quad c < \bar{c} \\ \{0, 1\} & \textbf{\textit{if}} \quad c = \bar{c} \\ 1 & \textbf{\textit{if}} \quad c > \bar{c} \end{cases}$$

where $\bar{c} = \frac{\beta^4 - 4\beta^2 - \beta + 4}{(\beta^2 - 2)^2}$.

(ii) Under Licensing and TAC holds, the cost inefficient public firm may be partially privatized.

(a) The Government will choose not to privatize if $c < \frac{\sqrt{-2\beta^2 + 2\beta + 4} - 2}{\sqrt{-2\beta^2 + 2\beta + 4}}$.

(b) The Government will choose to privatize partially at the level $c > \frac{\sqrt{-2\beta^2 + 2\beta + 4} - 2}{\sqrt{-2\beta^2 + 2\beta + 4}}$.

Therefore, in the mixed oligopoly where there is no technology transfer between the foreign private firm and domestic public firm, the government will privatize the public firm fully if the cost differential is too high and the government will not privatize at all if the cost differential is not significant. The threshold cost differential depends on the degree of product substitutability. We found no possibility of partial privatization as suggested by Ohnishi(2010).

But when we allow for technology transfer from the cost efficient foreign private firm to the cost inefficient public firm, we get the optimal response of the government as not to privatize at all if the cost differential is sufficiently low. However, when the cost differential is high enough then the government will choose to privatize partially. Here also, the threshold cost differential depends on the degree of product substitutability.

[5] The expression of θ^{LB} is provided in the Appendix.

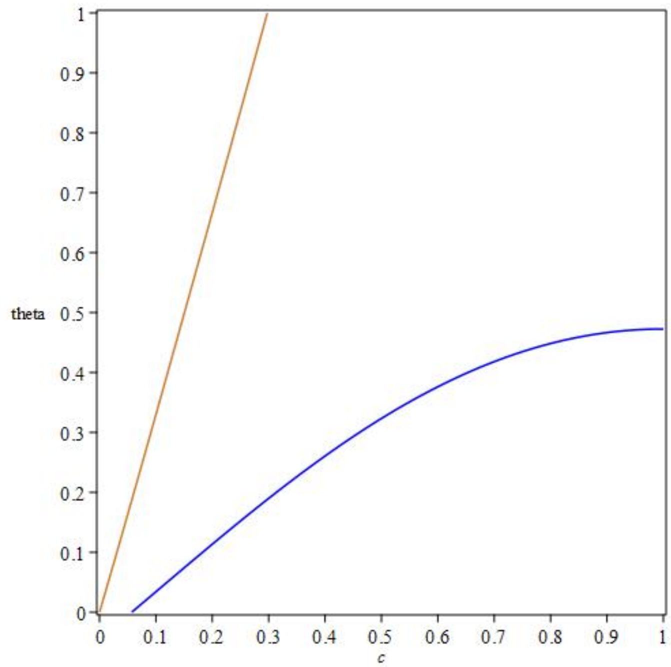


Figure 1: Optimal level of privatization(at $\beta = 0.5$) under licensing in Bertrand competition

In figure 1, the optimal level of privatization under Bertrand competition is shown, when the degree of product substitutability β is fixed at 0.5. In the left of the golden line, there will be no technology transfer as rationalizability condition fails to hold. The blue line cuts the horizontal axis at $c = 0.0572$(approx.) and up to that level of cost differential, the government will choose to not privatize at this given level of β. But when the cost differential is higher, the government will optimally choose to privatize partially, indicated by the blue line.

The major argument in favor of privatization is efficiency gain of the public firm. On the other hand privatization also puts less weight on consumer surplus and there will be deadweight loss. In the absence of licensing, privatization is the only way to achieve efficiency gain. If the public firm is not that inefficient then the efficiency gain from reduced marginal cost is outweighed by the loss in consumer surplus. Therefore it is optimal for the government not to privatize the public firm when the cost difference between the public firm and the private firm is not that much. But when the public firm is very inefficient then the efficiency gain outweighs the loss in consumer surplus and the government will fully privatize the public firm. Since price competition leads to lowering of the prices towards the marginal cost, the gain from reduced price is substantial and therefore full privatization does not hurt the consumers that much even if the goods the imperfect substitutes. This explains the government's push for full privatization, without licensing, when the public firm is sufficiently inefficient. At the threshold, when the 'switch' from no to full privatization takes place, both can be optimal implying that that the threshold the government is indifferent between the two. Given the possibility of licensing, there is an additional channel through which the public firm can achieve efficiency. Therefore, when the public firm is sufficiently inefficient, full privatization is no longer required from an efficiency point of view and the government can optimally go for partial privatization, hence keeping some weight on the consumer surplus as well. Additionally, it is noteworthy that the no privatization zone shrinks in the presence of licensing and the (partial) privatization zone increases. Since the licenser (private firm) extracts all the benefits through fixed costs when there is no privatization, the domestic welfare falls. Therefore, in the presence of licensing it is optimal for the domestic government to go for partial privatization for a larger range of inefficiency. Because of this the no privatization zone shrinks. But full privatization is no longer required in the presence of licensing.

4 Privatization under Cournot competition

4.1 Cournot competition under no licensing

Under no licensing Cournot competition the three stage game is reduced to a two stage game- (I) the Government decides the extent of privatization and (II) the both firms decide on quantity produced simultaneously and independently taking prices as given according to Cournot competition.

The profit functions of firm 1 and 2 are respectively as follows:

$$\Pi_1^{NC} = (p_1(q_1, q_2) - c)q_1 \tag{56}$$

$$\Pi_2^{NC} = p_2(q_1, q_2)q_2 \tag{57}$$

The social welfare in this case is

$$W(q_1, q_2) = CS(q_1, q_2) + \Pi_1^{NC}(q_1, q_2) \tag{58}$$

The objective function of the public firm(firm 1) is

$$V^{NC}(q_1, q_2) = \theta \Pi_1^{NC}(q_1, q_2) + (1 - \theta)W^{NC}(q_1, q_2) \tag{59}$$

Firm 1 will maximize its objective function to decide how much to produce. To maximize, after differentiating with respect to q_1 and equating to 0, we get the following first order condition:

$$\frac{\partial V^{NC}(q_1, q_2)}{\partial q_1} = \theta \frac{\partial \Pi_1^{NC}(q_1, q_2)}{\partial q_1} + (1 - \theta) \frac{\partial W^{NC}(q_1, q_2)}{\partial q_1} = 0 \tag{60}$$

Firm 2 will decide its production maximizing its Profit. We differentiate $\Pi_2^{NC}(q_1, q_2)$ with respect to q_2 and equate it with 0 to get the following first order condition:

$$\frac{\partial \Pi_2^{NC}(q_1, q_2)}{\partial q_2} = q_2 \frac{\partial p_2(q_1, q_2)}{\partial q_2} + p_2(q_1, q_2) = 0 \tag{61}$$

solving (60) and (61) for q_1 and q_2, we get

$$q_1^{\hat{N}C} = \frac{\theta\beta + 2c - 2}{\theta\beta^2 - 2\theta - 2} \tag{62}$$

$$q_2^{\hat{N}C} = \frac{-c\beta - \theta + \beta - 1}{\theta\beta^2 - 2\theta - 2} \tag{63}$$

Using (62) and (63) in equations (2), we get

$$p_1^{\hat{N}C} = \frac{(c+\theta-1)\beta^2 + \beta - 2c - 2\theta}{\theta\beta^2 - 2\theta - 2} \tag{64}$$

$$p_2^{\hat{N}C} = \frac{-c\beta - \theta + \beta - 1}{\theta\beta^2 - 2\theta - 2} \tag{65}$$

Using (62) and (63) in equations (56), (57), (4), (58) and (59) we get the following:

$$\Pi_1^{\hat{N}C} = \Pi_1^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C}) \tag{66}$$

$$\Pi_2^{\hat{N}C} = \Pi_2^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C}) \tag{67}$$

$$C\hat{S}^{NC} = CS(q_1^{\hat{N}C}, q_2^{\hat{N}C}) \tag{68}$$

$$W^{\hat{N}C} = W^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C}) \tag{69}$$

$$V^{\hat{N}C} = V^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C}) \tag{70}$$

Now, the Government will determine the level of privatization maximizing the social welfare. Differentiating $W^{\hat{N}C}$ with respect to θ, we get the following first order condition:

$$\frac{\partial W^{\hat{N}C}}{\partial \theta} = \frac{\partial W^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C})}{\partial q_1^{\hat{N}C}} \frac{\partial q_1^{\hat{N}C}}{\partial \theta} + \frac{W^{NC}(q_1^{\hat{N}C}, q_2^{\hat{N}C})}{\partial q_2^{\hat{N}C}} \frac{\partial q_2^{\hat{N}C}}{\partial \theta} = 0 \tag{71}$$

Solving (71), we get optimal level of privatization $\theta^{*NC}(c, \beta)$ as

$$\theta^{*NC}(c, \beta) = \frac{\beta(c\beta - \beta + 1)}{2c\beta^2 - 2\beta^2 - 4c + \beta + 4} \tag{72}$$

4.2 Cournot competition under licensing

For Cournot competition under licensing, the profit function of firm 1 and firm 2 becomes respectively

$$\Pi_1^{LC}(q_1, q_2) = (p_1(q_1, q_2) - \omega)q_1 - f \tag{73}$$

$$\Pi_2^{LC}(q_1, q_2) = p_2(q_1, q_2)q_2 + \omega q_1 + f \tag{74}$$

The social welfare function becomes

$$W^{LC}(q_1, q_2) = CS(q_1, q_2) + \Pi_1^{LC}(q_1, q_2) \tag{75}$$

The objective function of firm 1 in this case is

$$V^{LC}(q_1, q_2) = \theta\Pi_1^{LC}(q_1, q_2) + (1-\theta)W^{LC}(q_1, q_2) \tag{76}$$

At stage III, the firms will decide on how much to produce taking prices given according to Cournot setup. To maximize firm 1's objective function, differentiating $V^{LC}(q_1, q_2)$ with respect to q_1 and equating it with 0, we get the following first order condition:

$$\frac{\partial V^{LC}(q_1,q_2)}{\partial q_1} = \theta\frac{\partial \Pi_1^{LC}(q_1,q_2)}{\partial q_1} + (1-\theta)\frac{\partial W^{LC}(q_1,q_2)}{\partial q_1} = 0 \tag{77}$$

For maximization of firm 2's profit, we differentiate $\Pi_2^{LC}(q_1,q_2)$ with respect to q_2 and equate it with 0 to obtain the following first order condition:

$$\frac{\partial \Pi_2^{LC}(q_1,q_2)}{\partial q_2} = q_2\frac{\partial p_2(q_1,q_2)}{\partial q_2} + p_2(q_1,q_2) + \omega\frac{\partial q_1}{\partial q_2} = 0 \tag{78}$$

Using (77) and (78) we solve for q_1 and q_2, we get

$$q_1^{\hat{LC}} = \frac{\theta\beta\omega + \theta\beta + 2\omega - 2}{\theta\beta^2 - 2\theta - 2} \tag{79}$$

$$q_2^{\hat{LC}} = \frac{\theta\omega + \beta\omega + \theta - \beta + \omega + 1}{\theta\beta^2 - 2\theta - 2} \tag{80}$$

Putting (79) and (80) in (2), we get

$$p_1^{\hat{LC}} = \frac{(\theta+\omega-1)\beta^2 + \beta(\omega+1) - 2\theta - 2\omega}{\theta\beta^2 - 2\theta - 2} \tag{81}$$

$$p_2^{\hat{LC}} = \frac{(-\beta^2\omega + \omega - 1)\theta - (\omega-1)(\beta-1)}{\theta\beta^2 - 2\theta - 2} \tag{82}$$

Also using (79) and (80) in (73), (74), (4), (75) and (76), we get

$$\Pi_1^{\hat{LC}} = \Pi_1^{LC}(q_1^{\hat{LC}}, q_2^{\hat{LC}}) \tag{83}$$

$$\Pi_2^{\hat{LC}} = \Pi_2^{LC}(q_1^{\hat{LC}}, q_2^{\hat{LC}}) \tag{84}$$

$$C\hat{S}^{LC} = CS(q_1^{\hat{LC}}, q_2^{\hat{LC}}) \tag{85}$$

$$W^{\hat{LC}} = W^{LC}(q_1^{\hat{LC}}, q_2^{\hat{LC}}) \tag{86}$$

$$V^{\hat{LC}} = V^{LC}(q_1^{\hat{LC}}, q_2^{\hat{LC}}) \tag{87}$$

In stage II, the private firm(Firm 2) will decide the two-part tariff. The fixed fee (f) will be calculated based on the surplus in the objective function the public firm(Firm 1) enjoys under licensing contract than the no licensing scenario. Using (70) and (87) we calculate the fixed fee charged as:

$$f^{LC} = V^{\hat{LC}} - V^{\hat{NC}} = \frac{1}{(\theta\beta^2 - 2\theta - 2)^2}\sum_{i=0}^{4}\Phi_i(\omega,c,\beta)\theta^i \tag{88}$$

where $\Phi_3(\omega,c,\beta) = \omega(\beta^2-1)(\omega+2)$, $\Phi_2(\omega,c,\beta) = \omega(\omega+2)\beta^2 + 2(\omega^2-c)\beta - \omega(\omega+2)$, $\Phi_1(\omega,c,\beta) = (c-\omega)(c+\omega-2)\beta^2 + 4(\omega^2-c)\beta - 4c^2 + 5\omega^2 + 8c - 6\omega$ and $\Phi_0(\omega,c,\beta) = (\omega-c)(\omega+c-2)\beta^2 + 2(\omega^2-c)\beta - 4c^2 + 5\omega^2 + 8c - 6\omega$.

Putting (88) in equation (84), we get

$$\Pi_2^{\tilde{LC}} = \{\Pi_2^{\hat{LC}} | f = f^{LC}\} \tag{89}$$

Now, in order to maximize firm 2's profit, we differentiate $\tilde{\Pi_2^{LC}}$ with respect to ω and equate it with 0 to derive the first order condition. Solving the first order condition, we find the optimal ω as follows:

$$\omega^{*LC} = \frac{(1-\theta)((\beta^2-1)\theta^2 + 2\theta\beta^2 + 3\beta^2 + 1)}{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (-\beta^2+8\beta+9)\theta + 3\beta^2 + 6\beta + 7} \tag{90}$$

Putting (90) in (88), we get the final value of optimal fixed fee under licensing as

$$f^{*LC} = \{f^{LC} | \omega = \omega^{*LC}\} \tag{91}$$

Technology Adoption Condition: any rational firm will choose a contract only if its cost will reduce when under licensing contract compared to the situation with no-licensing. Hence, we impose the rationality assumption for firm 1, i.e., the cost-inefficient public firm will go for the contract only when the tariff rate under licensing is less than its original marginal cost under no-licensing.

$$\omega^{*LC} < c \tag{92}$$

Replacing (90) and (91) in (79), (80), (81), (82) we get,

$$\bar{q_1}^{LC} = \frac{2(\theta+\beta+1)(3-\theta)}{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (-\beta^2+8\beta+9)\theta + 3\beta^2 + 6\beta + 7} \tag{93}$$

$$\bar{q_2}^{LC} = \frac{2(\theta+\beta+1)(3-\theta)}{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (-\beta^2+8\beta+9)\theta + 3\beta^2 + 6\beta + 7} \tag{94}$$

$$\bar{p_1}^{LC} = \frac{(\beta^2-1)\theta^3 + (-\beta^2+2\beta+3)\theta^2 + (-3\beta^2+6\beta+5)\theta + 3\beta^2 - 4\beta + 1}{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (-\beta^2+8\beta+9)\theta + 3\beta^2 + 6\beta + 7} \tag{95}$$

$$\bar{p_2}^{LC} = \frac{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (\beta^2+2\beta+5)\theta - 3\beta^2 + 3}{(\beta^2-1)\theta^3 + (\beta+1)^2\theta^2 + (-\beta^2+8\beta+9)\theta + 3\beta^2 + 6\beta + 7} \tag{96}$$

Putting (93) and (94) in (83), (84), (85), (86) we get,

$$\bar{\Pi_1}^{LC} = \hat{\Pi_1^{LC}}(\bar{q_1}^{LC}, \bar{q_2}^{LC}) \tag{97}$$

$$\bar{\Pi_2}^{LC} = \hat{\Pi_2^{LC}}(\bar{q_1}^{LC}, \bar{q_2}^{LC}) \tag{98}$$

$$\bar{CS}^{LC} = \hat{CS^{LC}}(\bar{q_1}^{LC}, \bar{q_2}^{LC}) \tag{99}$$

$$\bar{W}^{LC} = \hat{W^{LC}}(\bar{q_1}^{LC}, \bar{q_2}^{LC}) \tag{100}$$

Finally, in stage I, the government will decide the extent of privatization, θ^{LC}, maximizing the social welfare. Using (100), differentiating $\bar{W}^{LC}$ with respect to θ and equating it with 0, we get the following first order condition,

$$\frac{\partial \bar{W}^{LC}(\bar{q_1}^{LC}, \bar{q_2}^{LC})}{\partial \theta} = \frac{\partial \bar{W}^{LC}(\bar{q_1}^{LC}, \bar{q_2}^{LC})}{\partial \bar{q_1}^{LC}} \frac{\partial \bar{q_1}^{LC}}{\partial \theta} + \frac{\partial \bar{W}^{LC}(\bar{q_1}^{LC}, \bar{q_2}^{LC})}{\partial \bar{q_1}^{LC}} \frac{\partial \bar{q_1}^{LC}}{\partial \theta} = 0 \tag{101}$$

solving (101), we get $\theta^{LC}(c,\beta)$ from which we obtain the optimal level of privatization under licensing in Cournot competition as $\theta^{*LC}(c,\beta)$. [6]

$$\theta^{*LC}(c,\beta) = max[\theta^{LC}(c,\beta), 1] \tag{102}$$

We now summarize our finding in the above two subsection under Cournot competition in the following proposition.

Proposition 2. *Under Cournot competition, the following holds:*

(i) The cost inefficient public firm will decide to privatize partially while no licensing. The optimal level of privatization $\theta^{*NC}(c,\beta)$ is

$$\theta^{*NC}(c,\beta) = \frac{\beta(c\beta - \beta + 1)}{2c\beta^2 - 2\beta^2 - 4c + \beta + 4}$$

.

(ii) Under Licensing and TAC holds,

(a) The cost inefficient public firm will be partially privatized if the cost differential is sufficiently low. The Government will choose to privatize partially at the level $\theta^{LC}(c,\beta)$ if $c < \frac{(\beta-2)(\beta^3-2\beta^2+4\beta+8-\sqrt{3\beta^6+2\beta^5+32\beta^4+48\beta^3-16\beta^2-32\beta})}{\beta^4+8\beta^2-16}$.

(b) The cost inefficient public firm will be fully privatized if the cost differential is high enough. The Government will choose to privatize fully if $c \geq \frac{(\beta-2)(\beta^3-2\beta^2+4\beta+8-\sqrt{3\beta^6+2\beta^5+32\beta^4+48\beta^3-16\beta^2-32\beta})}{\beta^4+8\beta^2-16}$.

Hence, we can say the results of the optimal privatization level in case of a cost-reducing technology transfer where the foreign private firm is transferring its superior technology to the inefficient public firm is robust with the existing literature. Even if we consider a different technology(,i.e., constant returns to scale technology), technology transfer under different licensing mechanism (,i.e., two-part tariff mechanism) and differentiated product, we get partial privatization is optimal for Cournot competition.

In the figure 2, the cyan line represents the minimum level of privatization that is required for rationalizability condition to hold for different levels of cost differential when the degree of product substitutability is given at 0.1. The red line gives the optimal level of privatization for different cost differential levels given $\beta = 0.1$. At the left of the intersection point between the two lines, the licensing contract will not happen.

Since quantity competition is not that stringent the effective price reduction is less under Cournot compared to Bertrand. So the consumers does not gain much and there is relative welfare loss under Cournot. To compensate for that the domestic government will always go for some privatization, viz. partial privatization and no privatization is not an option anymore, in the absence of technology licensing. This holds for all levels of inefficiency of the domestic public firm. In the presence of licensing, there is an additional tool to achieve efficiency. But the foreign firm charges a per-unit license fee that effectively increases the MC of the domestic firm. Also the foreign firm extracts all the surplus from the domestic firm through the fixed cost. That reduces the domestic welfare.

[6]The expression of θ^{LC} is provided in the Appendix.

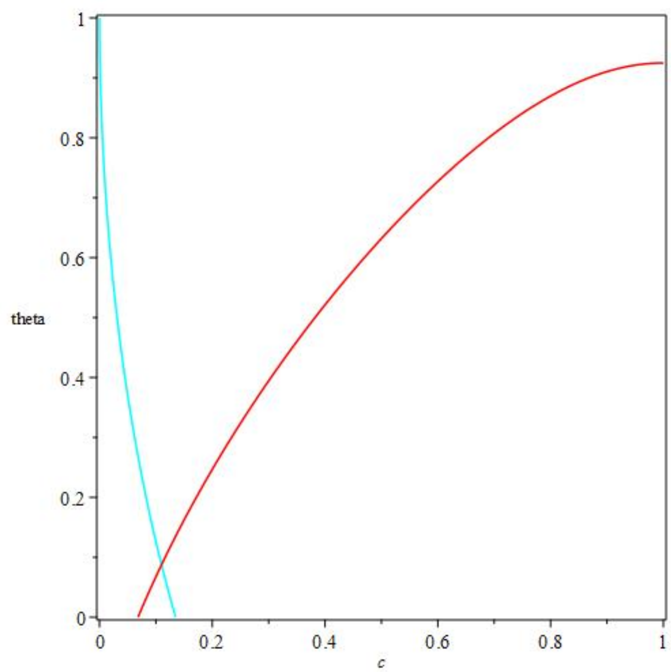


Figure 2: Optimal level of privatization (at $\beta = 0.1$) under licensing in Cournot competition

To counter this, the domestic government will fully privatize the domestic public firm if it is too inefficient. If it is not that inefficient it will partially privatize it. But no privatization is never an option under Cournot competition.

5 Technology Adoption condition

In this section, we shall take a deeper look on technology adoption condition. In two-part tariff licensing, the licenser charges the licensee a combination of a per-unit royalty fee (ω) and a fixed fee (f) in such a manner that the licensee achieves the cost efficiency (,i.e., the cost differential vanishes) but the licenser will extract all the surplus gained by the licensee due to the elimination of cost inefficiency. If the licenser charges per-unit royalty fee more than the cost differential, then any rational firm will not engage in such licensing contract. So, by the same rationale as considered by Chen et al.(2014), we are assuming that any licensing contract holds if and only if the post-licensing per unit royalty fee (ω^{*L}) is less than the pre-licensing cost differential (c) under both the condition.

In our analysis, we found that the technology adoption condition in our model not only depends on the supply side parameter, the cost differential (c) as suggested by the existing literature (Chen et al.(2014)) but it depends on the demand side parameter, the degree of product substitutability (β) also. The technology adoption condition imposes that for a given level of the degree of product substitutability the pre-licensing cost differential must be high enough such that the publicly regulated firm becomes willing to acquire the superior technology through two-part tariff licensing. In case of Bertrand competition, licensing can occur for every (c, β) optimally. But in case of Cournot competition, licensing cannot occur for low cost differential and low degree of product substitutability. In that case, the government set the optimal privatization level at no-licensing optimal privatization level. In Figure 1, we can see that the licensing is feasible for every level optimal privatization given

$\beta = 0.5$, and the technology adoption condition is non-restrictive for Bertrand competition. But in 2, we can observe that the licensing is not feasible at the optimal level of privatization for low level of pre-licensing cost differential (say, at $c = 0.08$) given $\beta = 0.1$ under Cournot competition.

We can summarize our findings in the following proposition:

Proposition 3. *When a foreign private firm licenses its cost-efficient technology to a publicly regulated firm via two-part tariff mechanism in a mixed oligopoly, the rationalizability condition imposes the following restrictions:*

(i) In case of Bertrand competition, at optimal privatization, technology adoption condition is non-restrictive.

(ii) In case of Cournot competition, for any given β, there exists a threshold $\tilde{c}^{LC}$ such that the two-part tariff licensing will hold only when the pre-licensing cost-differential is greater than that threshold level, i.e., $c > \tilde{c}^{LC}(\beta)$. At optimum, this condition fails to satisfy when both the cost differential and the degree of product substitutability is at low level.

6 Comparison between Bertrand and Cournot competition

In the earlier sections we have found that partial privatization is possible under two-part tariff licensing in case of Bertrand competition in mixed oligopolies if the cost differential is high enough and technology adoption condition holds. In similar framework, Cournot competition also yields partial or full privatization as optimal privatization decision.

Now, we can compare the optimal privatization decision under Cournot competition and Bertrand competition in the common region where technology adoption condition holds for both the competition. Comparing both type of competition, we found that the degree of product substitutability and the cost differential being low enough, the optimal level of privatization under Bertrand competition is higher than that under Cournot competition when the cost-efficient foreign private firm is transferring its superior technology to the cost-inefficient public firm via two-part tariff in a mixed oligopoly. But when the degree of product substitutability and the cost differential gets sufficiently high, the ranking of the optimal privatization level got reversed, i.e., the optimal privatization under Cournot competition is higher than that under Bertrand competition. The comparison is stated in the following proposition.

Proposition 4. *under licensing and TAC holds, The optimal level of privatization in case of Bertrand competition is higher compared to that in case of Cournot competition for the region where the cost differential and the degree of product substitutability is sufficiently low. Beyond that level, the ranking got reversed and the optimal level of privatization is higher in case of Cournot competition compared to that under Bertrand competition.*

In the figure 3, we have plotted the optimal degree of privatization under Cournot and Bertrand competition keeping the degree of product substitutability fixed at 0.06. The red and blue line gives the optimal degree of privatization for different cost differentials under Cournot and Bertrand competition respectively. The green line gives the optimal level of privatization for Cournot competition under no licensing. The golden line represents the technology adoption condition for Bertrand competition and the cyan line represents the technology adoption condition for Cournot competition.

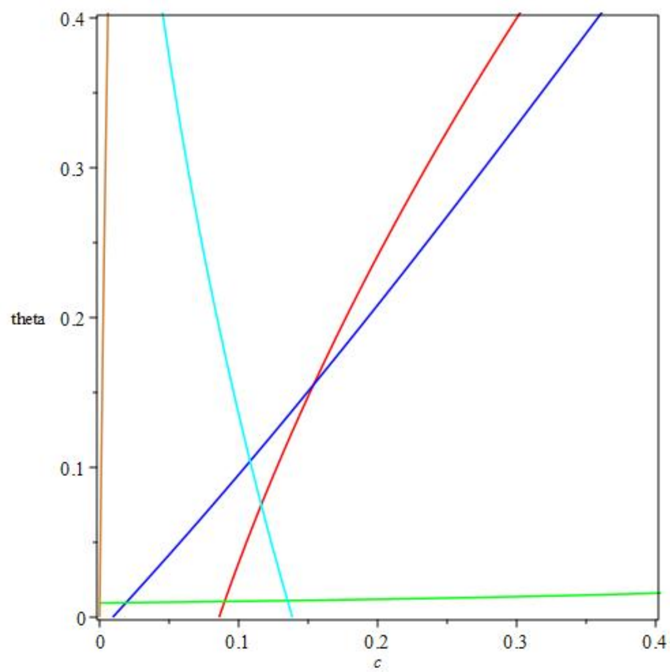


Figure 3: Cournot-Bertrand comparison of optimal level of privatization under licensing (at $\beta = 0.06$)

We can observe the technology adoption condition insists that there will be no licensing contract at the optimal level of privatization under Cournot competition for low cost differential (left of the cyan line). Under licensing and RC, for low enough cost differential the Bertrand level of optimal privatization is higher that the Cournot level of optimal privatization. But for higher cost differential the ranking got reversed, i.e., the optimal degree of privatization under Cournot competition is higher than that under Bertrand competition.

When products are not close substitutes, price competition is relaxed and therefore the prices do not fall much implying that the CS does not increase that much. So when the domestic firm is not that inefficient, and also the prices do not fall that much, higher privatization is needed to achieve economic efficiency and this incentive is greater for Bertrand than under Cournot. So the 'price competition effect' in increasing CS is lower for lower level of inefficiency and that's why greater privatization is needed in case of price competition. But as inefficiency of the domestic firm is higher, the 'price competition effect' in increasing CS increases and in that case not much privatization is needed under Bertrand. On the contrary more privatization is required in case of Cournot competition to achieve efficiency compared to Bertrand. This non-monotonicity holds when products are not close substitutes.

7 Product substitutability and privatization

In the previous section, we found that the optimal degree of privatization not only depends on supply side parameters like cost differential but on demand side parameter the degree of product substitution also. Now, we shall focus on how the optimal level of privatization(θ) changes with the degree of product substitution(β) and the cost differential(c). We have found that keeping the degree of product substitution fixed in a specific level, the optimal degree of privatization under two-part tariff private licensing from a foreign firm in a mixed oligopoly varies proportionately with the cost differential under both type of competition. The optimal degree of (partial)privatization increases as the cost differential is rises. And this behavior between θ and c is consistent under the both type of competition. This is quite intuitive. As efficiency enhancement through cost reduction is the goal of such privatization, then we can expect the government will privatize more if the cost differential is higher.

Remark. *In a mixed duopoly where a foreign private firm is licensing its technology to the publicly regulated firm via two-part tariff mechanism, the optimal degree of privatization is increasing in cost differential, keeping the degree of product differentiation fixed for both type of competition.*

From this above remark, it becomes clear that the difference between the optimal privatization decisions under Cournot and Bertrand competition does not comes from the cost differential; it is essentially same story for the both type of competition if we focus on the cost differential keeping the degree of product substitutability fixed. And that motivate us to focus on the degree of product substitutability as the pivotal factor.

What we have found investigating the relationship between the optimal degree of privatization and the degree of product substitutability while keeping the cost differential fixed is stated in the next proposition.

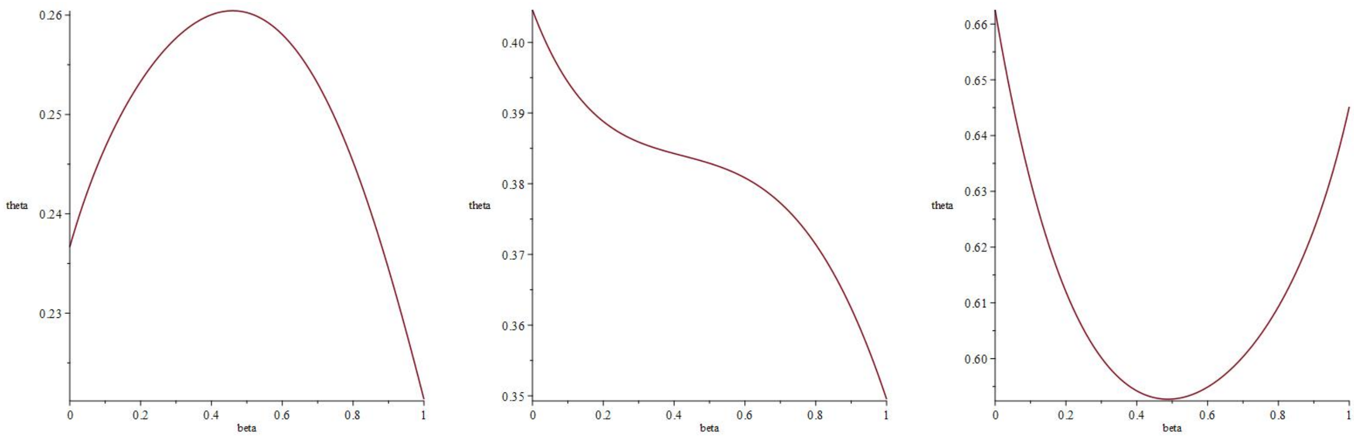
Proposition 5. *With TAC and licensing contract, the following relation between the optimal level of privatization and the degree of product substitution holds:*

(i) Under Cournot competition, for a given cost differential is sufficiently low, i.e., $c < \bar{c}_{C1}$, the optimal degree of privatization (θ^{*LC}) and the degree of product substitutability (β) are related in inverted-U manner. However, if the cost differential is given at high enough level, i.e., $c > \bar{c}_{C2}$, the relation between the two becomes U-shaped.

(ii) Under Bertrand competition for a given cost differential less than $\bar{c}_B$, the optimal degree of privatization (θ^{*LB}) and the degree of product substitutability (β) are related in U-shaped manner. However, if the cost differential is given at high enough level, i.e., $c > \bar{c}_B$, the optimal degree of privatization decreases with the degree of product substitutability.

Fujiwara(2007) found that the optimal degree of privatization in a mixed oligopoly under Cournot competition varies in inverted-U manner with the degree of product substitutability for a given cost differential. However, when we introduce licensing via two-part tariff in this context, we have found that the inverted-U relationship holds if the cost differential is low enough. That is, if the cost differential is given at a low enough level, the optimal degree of privatization initially increases as the degree of product substitutability increases up to a certain level, but thereafter the optimal degree of privatization falls as the degree of product substitutability rises. If the cost differential is sufficiently high, then the relationship becomes U-shaped. Thus, if the cost differential is given beyond a threshold level, then the optimal degree of privatization initially falls with increasing degree of product substitutability up to a certain level and beyond that level of product substitutability, as it rises the optimal degree of privatization falls.

In the figure 4 we have shown the relationship between the optimal degree of privatization and the degree of product substitutability under Cournot competition in case of licensing via two-part tariff mechanism in a mixed oligopoly where the foreign private firm is the licenser. In figure 1(a),



(a) $c = 0.2$ (b) $c = 0.3$ (c) $c = 0.5$

Figure 4: The relation between optimal degree of privatization and the degree of product differentiation under Cournot competition

the inverted-U relationship holds when the cost differential is low ($c = 0.2$). When the cost differential, $c = 0.3$, we found a overall decreasing optimal degree of privatization with degree of product substitutability. But with high cost differential ($c = 0.5$), we derive U-shaped relationship between the two.

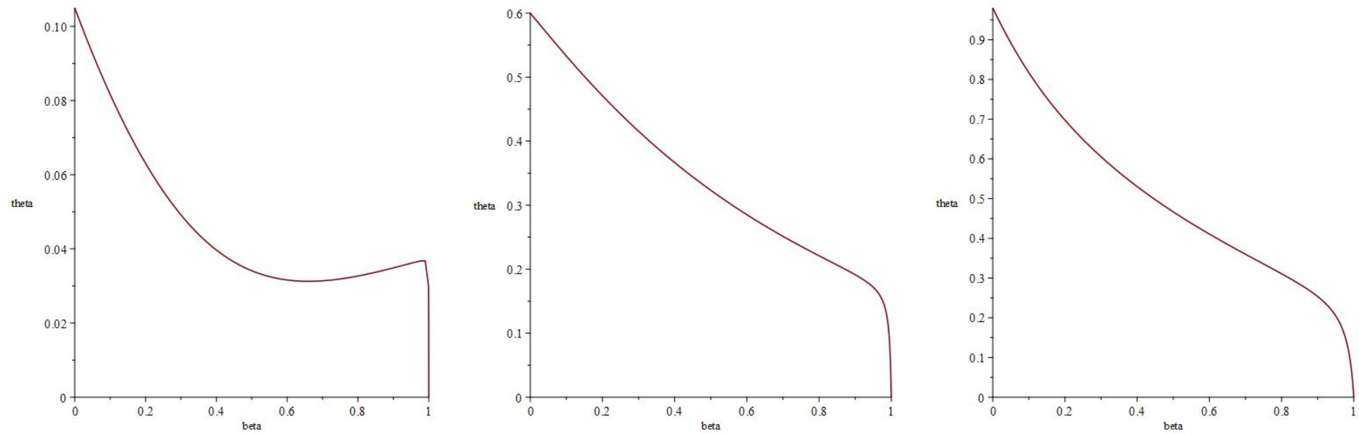
We have also computed the relationship under Bertrand competition with licensing. We found that at a given level of cost differential, the optimal degree of privatization and the degree of product substitutability is inverted-U shaped if the cost differential is fixed at a low level. But the optimal degree of privatization and the degree of product substitutability varies in decreasing manner if the given cost differential is sufficiently high. That is, if the cost differential is low, the optimal degree of privatization falls with the degree of product substitutability up to a certain level of product substitutability and beyond that level of product substitutability, it rises again. If the cost differential is high enough, then the optimal degree of privatization falls throughout when the degree of product substitutability rises.

In the Figure 5, we have shown the relationship between the optimal degree of privatization and the degree of product substitutability at different levels of cost differential in our framework under Bertrand competition. In Figure 5(a), we can observe the inverted-U relationship between the two when the cost differential is low, fixed at a level $c = 0.1$). Figure 5(b) and Figure 5(c) show the decreasing relationship between them when the cost differential is fixed at levels high enough.

8 conclusion

Keeping the gap in the production technology between the developed and developing economies in mind, serious attention is demanded on technology transfer mechanism and its interplay with welfare-driven privatization decisions of the Government of developing countries. Our study theoretically shades light on sustainable and inclusive technological transformation for developing countries and provides policy implications.

In this paper, we have studied the optimal privatization decision in a mixed oligopoly where technology transfer takes place from a cost-efficient foreign private firm to a cost-inefficient domestic



(a) $c = 0.1$ (b) $c = 0.5$ (c) $c = 0.9$

Figure 5: The relation between optimal degree of privatization and the degree of product differentiation under Bertrand competition

public firm via two-part tariff licensing. We found that privatization alone is not sufficient if the cost differential is too high, licensing provides another channel of inefficiency reduction and the degree of product substitutability affects the licensing decision also. Secondly, Unlike full privatization and no privatization binary in case of price competition suggested by earlier literature, partial privatization is possible if the cost differential is sufficiently high. Thirdly, we have partial or full privatization based on the cost differential under quantity competition as existing literature even with constant returns to scale technology, product differentiation and two-part tariff licensing.

We compared the optimal privatization decision under both type of competition and found that when both the cost differential and the degree of product substitutability is sufficiently low, the Bertrand level of optimal privatization is higher than the Cournot level. But it got reversed when the cost differential and the degree of product substitutability is sufficiently high. We have also investigated the relation between the optimal degree of privatization and the degree of product substitutability for a given cost differential and found that it is inverted-U shaped if the given cost differential is sufficiently low and U-shaped when the given cost differential is high enough in case of Cournot competition. For Bertrand competition, the relationship between the two is initially U-shaped and then decreasing as the given cost differential level increases.

Our paper can be further extended to investigate few deeper questions. An interesting insight will be the comparison between the optimal degree of privatization between no-licensing and licensing situation under both the competition and choice of privatization in case multiple equilibria, if exists. A further investigation can be done endogenizing the licensing regime. One possible limitation can be the following. In this model we have considered that the publicly regulated firm is sufficiently equipped for the technology transfer, i.e., it has enough resource to fund the fixed fee and the cost associated with installation of new technology and training its employees. The publicly regulated firm, even if it is not profitable, can have the fund via government transfer and the government collects the money imposing lumpsum taxes on the firms and the consumers and thus it does not affects the social welfare and the optimal privatization decision by the firms. However, this may not be true in reality as many of the developing countries are highly tax-burdened.

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9 Appendix

9.1 First Order Condition for Bertrand Competition with Licensing

$\frac{\partial W(c,\beta,\theta^{LB})}{\partial\theta} = 0 \implies \frac{\sum_{i=1}^{10}\Phi_i^{LB}(c,\beta)\theta^i}{2(\beta+1)(\theta+1)^3(-2+(\beta^2-2)\theta)^3(\beta-1)(\theta^2\beta^2+4\theta\beta^2+4)^2} = 0.$

where $\Phi_{10}^{LB}(c,\beta) = (-\beta^8 + 3\beta^6 - 2\beta^4)$,

$\Phi_9^{LB}(c,\beta) = (-11\beta^8 + 39\beta^6 - 28\beta^4)$,

$\Phi_8^{LB}(c,\beta) = 2(-12 - \beta^{10}/2 + (c^2 - 2c + 5)\beta^8 - c\beta^7 + (-9/2\beta^2 + 9c - 75/2)\beta^6 + 6\beta^5 + (7c^2 - 14 * c + 116)\beta^4 - 12\beta^3 + (-4c^2 + 8c - 73)\beta^2 + 8\beta)\beta^2$,

$\Phi_7^{LB}(c,\beta) = (\beta^{12} - 8\beta^{11} + (22c^2 - 44c + 24)\beta^{10} + (-22c + 72)\beta^9 + (-101c^2 + 202c - 207)\beta^8 - 96\beta^7 + (164c^2 - 328c + 706)\beta^6 + 40\beta^5 + (-96c^2 + 192c - 356)\beta^4 + 32\beta^3 - 208\beta^2)$,

$\Phi_6^{LB}(c,\beta) = (8\beta^{12} - 16\beta^{11} + (86c^2 - 172c + 19)\beta^{10} + (-86c + 240)\beta^9 + (-393c^2 + 786c - 268)\beta^8 + (-16c - 432)\beta^7 + (640c^2 - 1280c + 1151)\beta^6 + 416\beta^5 + (-320c^2 + 640c - 230)\beta^4 + 16\beta^3 + (-64c^2 + 128c - 840)\beta^2 - 64\beta)$,

$\Phi_5^{LB}(c,\beta) = (16\beta^{11} + (146c^2 - 292c + 75)\beta^{10} + (-146c + 128)\beta^9 + (-631c^2 + 1262c - 214)\beta^8 + (-112c - 104)\beta^7 + (932c^2 - 1864c + 759)\beta^6 + 304\beta^5 + (-64c^2 + 128c + 1212)\beta^4 + 352\beta^3 + (-512c^2 + 1024c - 2144)\beta^2 - 384\beta)$,

$\Phi_4^{LB}(c,\beta) = (112(c-1)^2\beta^{10} + (-112c + 48)\beta^9 + (-410c^2 + 820c - 204)\beta^8 + (-240c + 340)\beta^7 + (354c^2 - 708c + 30)\beta^6 + (-32c + 72)\beta^5 + (1064c^2 - 2128c + 2826)\beta^4 + 496\beta^3 + (-1184c^2 + 2368c - 2760)\beta^2 - 704\beta - 128c^2 + 256c - 256)$,

$\Phi_3^{LB}(c,\beta) = (32(c-1)^2\beta^{10} + (-32c + 32)\beta^9 + (-48c^2 + 96c - 36)\beta^8 + (-208c + 216)\beta^7 + (-296c^2 + 592c - 368)\beta^6 + (-96c + 392)\beta^5 + (1616c^2 - 3232c + 2724)\beta^4 - 224\beta^3 + (-960c^2 + 1920c - 1488)\beta^2 - 512\beta - 512c^2 + 1024c - 768)$,

$\Phi_2^{LB}(c,\beta) = 32(c-1)^2\beta^8 + (-64c + 64)\beta^7 + (-240c^2 + 480c - 280)\beta^6 + (-96c + 304)\beta^5 + (832c^2 - 1664c + 1424)\beta^4 - 528\beta^3 + (64c^2 - 128c - 56)\beta^2 - 192\beta - 768(c-1)^2$,

$\Phi_1^{LB}(c,\beta) = -32(c-1)^2\beta^6 + (-32c + 32)\beta^5 + (80c^2 - 160c + 288)\beta^4 - 160\beta^3 + (448c^2 - 896c + 256)\beta^2 - 128\beta - 512c^2 + 1024c - 256$,

and $\Phi_0^{LB}(c,\beta) = -32(\beta+2)(\beta-1)((c-1)^2\beta^2 - (c-1)^2\beta - 2c^2 + 4c)$.

We can find $\theta^{LB}(c,\beta)$ by solving the above equation.

9.2 First Order Condition for Cournot Competition with Licensing

$\frac{\partial W(c,\beta,\theta^{LC})}{\partial\theta} = 0 \implies \frac{\sum_{i=1}^{12}\Phi_i^C(c,\beta)\theta^i}{2(-2+(\beta^2-2)\theta)^3((\beta^2-1)\theta^3+(\beta+1)^2\theta^2+(-\beta^2+8\beta+9)\theta+3\beta^2+6\beta+7)^3} = 0.$

where $\Phi_{12}^{LC}(c,\beta) = (\beta^2-2)(\beta-1)^4(\beta+1)^4$,

$\Phi_{11}^{LC}(c,\beta) = 3\beta(\beta^2+\beta-4)(\beta-1)^3(\beta+1)^4$,

$\Phi_{10}^{LC}(c,\beta) = (\beta-1)^2((c^2-2c+5)\beta^6+(-4c+40)\beta^5+(-7c^2+14c-6)\beta^4+(4c-124)\beta^3+(14c^2-28c-47)\beta^2+60\beta-8c^2+16c+20)(\beta+1)^2$,

$\Phi_9^{LC}(c,\beta) = (\beta-1)(16\beta^8+(c^2-2c-47)\beta^7+(5c^2-26c+9)\beta^6+(-21c^2+34c+362)\beta^5+(-21c^2+58c-338)\beta^4+(52c^2-96c-843)\beta^3+(32c^2-64c+517)\beta^2+(-32c^2+64c+540)\beta-16c^2+32c-240)(\beta+1)^2$,

$\Phi_8^{LC}(c,\beta) = -6(\beta-1)(\beta+1)(8\beta^9+(c^2-2c+12)\beta^8+(-4c^2+10c-24)\beta^7+(-7c^2+42c-181)\beta^6+(38c^2-44c-130)\beta^5+(47c^2-106c+539)\beta^4+(-70c^2+122c+664)\beta^3+(-89c^2+178c-213)\beta^2+(24c^2-48c-490)\beta+36c^2-72c-137)$,

$\Phi_7^{LC}(c,\beta) = 4(-24\beta^{10}+(-159/2+c^2-2c)\beta^9+(-4c^2+4c+405/2)\beta^8+(16c^2-124c+1613/2)\beta^7+(-171/2-74c^2+24c)\beta^6+(-5751/2-185c^2+438c)\beta^5+(-3683/2+206c^2-300c)\beta^4+(6949/2+404c^2-800c)\beta^3+(6385/2-100c^2+200c)\beta^2+(-224c^2+448c-1218)\beta-16c^2+32c-1456)(\beta+1)$,

$\Phi_6^{LC}(c,\beta) = 10(\beta+1)(-16\beta^{10}+(-153/5+c^2-2c)\beta^9+(-47/5c^2+10c+841/5)\beta^8+(-2/5c^2-144/5c+479)\beta^7+(-951/5+114/5c^2-1124/5c)\beta^6+(-9877/5-1013/5c^2+1178/5c)\beta^5+(-6399/5-1089/5c^2+2702/5c)\beta^4+(2343+2378/5c^2-4176/5c)\beta^3+(11393/5+2502/*c^2-5004/5c)\beta^2+(-1048/5c^2+2096/5c-2896/5)\beta-232c^2-3684/5+464c)$,

$\Phi_5^{LC}(c,\beta) = (144\beta^{11}+(-42c^2+84c+318)\beta^{10}+(48c^2-144c+348)\beta^9+(-90c^2-444c+1716)\beta^8+(-888c^2-2112c+1476)\beta^7+(-3438c^2-1956c-11580)\beta^6+(-7728c^2+8688c-27180)\beta^5+(-894c^2+2028c-8244)\beta^4+(16344c^2-31200c+28404)\beta^3+(14736c^2-29472c+33918)\beta^2+(-480c^2+960c+19944)\beta-2976c^2+5952c+6816)$,

$\Phi_4^{LC}(c,\beta) = (144\beta^{11}+(20c^2-40c+557)\beta^10+(-36c^2+64c-760)\beta^9+(-624c^2-48c-3242)\beta^8+(-2536c^2+8c+1680)\beta^7+(-4448c^2-4096c+8482)\beta^6+(-8748c^2-2208c-14680)\beta^5+(-12552c^2+11232c-45760)\beta^4+(6616c^2-16616c-4056)\beta^3+(33876c^2-67752c+75113)\beta^2+(27744c^2-5548*c+79560)\beta+6768c^2-13536c+25842)$,

$\Phi_3^{LC}(c,\beta) = ((20c^2-40c+299)\beta^{10}+(-372c^2+568c-34)\beta^9+(-904c^2+848c-2978)\beta^8+(-1624c^2-1840c-1462)\beta^7+(-6524c^2-4648c+9104)\beta^6+(-18580c^2+2456c-2850)\beta^5+(-27936c^2+23328c-38022)\beta^4+(-992c^2-9248c+1182)\beta^3+(56400c^2-112800c+95885)\beta^2+(65088c^2-130176c+95740)\beta+22464c^2-44928c+27456)$,

$\Phi_2^{LC}(c,\beta) = (-99(c-1)^2\beta^{10}+(36c^2-108c+72)\beta^9+(-285c^2-6c-1320)\beta^8+(-3360c^2+732c-2412)\beta^7+(-12783c^2+4734c+3462)\beta^6+(-26244c^2+15900c+7884)\beta^5+(-24591c^2+15294c+684)\beta^4+(10944c^2-34572c+16044)\beta^3+(61542c^2-123084*c+56997)\beta^2+(67776c^2-135552c+51564)\beta+25368c^2-50736c+14580)$,

$\Phi_1^{LC}(c,\beta) = (81(c-1)^2\beta^{10}-54(c-1)^2\beta^9+(-1341c^2+1602c-342)\beta^8+(-5400c^2+5256c-882)\beta^7+(-11577c^2+8898c+1500)\beta^6+(-14862c^2+9036c+6186)\beta^5+(-6347c^2-4610c+8854)\beta^4+(16332c^2-39328c+8698)\beta^3+(36672c^2-73344c+11043)\beta^2+(34608c^2-69216c+9988)\beta+13328c^2-26656c+4080)$,

and $\Phi_0^{LC}(c,\beta) = -54(\beta^2+2\beta+7/3)((c-1)^2\beta^8+(4c^2-6c+2)\beta^7+(8/3+29/3c^2-34/3c)\beta^6+(40/3c^2-28/3c)\beta^5+(-8/9+91/9c^2-14/9c)\beta^4+(-110/9-20/3c^2+218/9c)\beta^3+(44/9-263/9c^2+526/9c)\beta^2+(-112/3c^2+224/3c+20/3)\beta-(196c^2)/9+(392c)/9-37/9)$.

We will find $\theta^{LC}(c,\beta)$ by solving the above equation.

9.3 Proof of Proposition 1

Proof of Proposition 1(i)

From equation (12), we derive the expression of $\theta^{NB}(c,\beta)$ as

$$\theta^{NB}(c,\beta) = \frac{\beta(1-\beta(1-c))}{(c-1)(\beta^2-2)^2+\beta}$$

As $\beta \in (0,1)$ and $c \in (0,1)$, the numerator is always strictly positive.The denominator is negative when $c < \bar{c}$ and greater than 1 when $c > \bar{c}$, where $\bar{c} = \frac{\beta^4-4\beta^2-\beta+4}{(\beta^2-2)^2}$. We find the expression of $\bar{c}$ after equating the denominator with 0 and a little algebraic manipulation.

Thus, the government will set the optimal privatization level at $\theta*^{NB}$ at 0 when the cost differential (c) is less than the cut-off $\bar{c}(\beta)$ and at 1 when the cost differential (c) is greater than the cut-off $\bar{c}(\beta)$. Hence, proved.

Proof of Proposition 1(ii)

From equation (30) we get the first order condition described at section 9.1. By setting $\theta*^{LB} = 0$ in that expression, the first order condition is reduced to

$$\frac{1}{4}((c-1)^2(\beta^2-\beta-1)-2c(c-1)) = 0$$

Simplifying and considering only the roots in the assumed domain of c, we get $\bar{c}^{LB} = \frac{\sqrt{-2\beta^2+2\beta+4}-2}{\sqrt{-2\beta^2+2\beta+4}}$. For all pairs of (c,β) where $c < \bar{c}^{LB}$, the optimal value of $\theta *^{LB}(c,\beta)$ will be negative and hence the government will not privatize for that case. Keeping β at a fixed level, $\theta *^{LB}(c,\beta)$ is increasing in c. Also, as $\beta \to 0$ and $c \to 1$, $\theta *^{LB}(c,\beta) \to 1$. Thus, for $c \in (0,1)$ and $\beta \in (0,1)$, when $c > \bar{c}^{LB}$, we get $\theta *^{LB}(c,\beta) \in (0,1)$, i.e., partial privatization. Hence, Proved.

9.4 Proof of Proposition 2

Proof of Proposition 2(i)

From (72), we get the optimal level of privatization for no licensing condition under Cournot competition. It can be easily shown that $\theta*^{NC} \in (0,1) \forall c \in (0,1) \forall \beta \in (0,1)$.

Proof of Proposition 2(ii)

In section 9.2, we provide the first order condition solving which we can get $\theta *^{LC}(c,\beta)$. If we set $\omega = \omega*^{LC} = c$ form equation 90 in that first order condition, then the first order condition reduces to a function of β and ω at the lower boundary points such that technology adoption condition holds. If $c \geq \frac{(\beta-2)(\beta^3-2\beta^2+4\beta+8-\sqrt{3\beta^6+2\beta^5+32\beta^4+48\beta^3-16\beta^2-32\beta})}{\beta^4+8\beta^2-16}$, solving the first order condition we get $\theta*^{LC} \geq 1$. Thus, the optimal privatization for the government is full privatization. For $\tilde{c}^{LC} \leq c < \frac{(\beta-2)(\beta^3-2\beta^2+4\beta+8-\sqrt{3\beta^6+2\beta^5+32\beta^4+48\beta^3-16\beta^2-32\beta})}{\beta^4+8\beta^2-16}$, we get $0 < \theta*^{LC} < 1$, i.e., partial privatization. Hence, proved.

9.5 Proof of Proposition 3

Proof of Proposition 3(i) Equation (54) and the equation $\omega *^{LB}(c,\beta) = c$ intersects only when $\beta = 0$, which is not possible due to restriction in the domain of β.

Proof of Proposition 3(ii) From equation (90), we get $\omega *^{LC}(c,\beta) = \omega *^{LC}(\beta, \theta *^{LC}(c,\beta))$. We can calculate the threshold cost differential, $\tilde{c}^{LC}$, from the expression $\omega *^{LC}(c,\beta)$ for a given β. We simulate the value of $\tilde{c}^{LC}$ for different values of β and θ in Table 1.

To investigate whether this condition restricts licensing under optimal level of privatization, we simulate $\theta*^{LC}$ for different values of c and β in Table 2 and Table 3 and found that the rationalizability condition restricts licensing under the optimal level of privatization for low enough c and β (for example, at $c = 0.1$ and $\beta = 0.1$) for Cournot competition.

9.6 Proof of Proposition 4

To prove our proposition, we simulate $\theta*^{LC}$ and $\theta*^{LB}$ for different values of c and β in Table 2 and compare them. The shaded rows in Table 2 give $\theta*^{LB}$ for a given level of c under Bertrand competition and non-shaded rows give $\theta*^{LC}$ for a given level of of c under Cournot competition. The blank cells indicate the violation of rationalizability condition under Cournot competition. The green cells indicates the pair of (c,β) for which the optimal degree of privatization under Bertrand competition is higher than that under Cournot competition. This is true only when the both c and β is sufficiently low. In other cases, the the optimal degree of privatization under Bertrand competition is lower than that under Cournot competition.

$\theta \backslash \beta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	0.134993	0.134615	0.140022	0.149798	0.162791	0.178082	0.194949	0.212828	0.231288
0.1	0.106714	0.107201	0.112656	0.121883	0.133916	0.147984	0.163480	0.179929	0.196959
0.2	0.082622	0.083771	0.089155	0.097763	0.108787	0.121582	0.135641	0.150564	0.166037
0.3	0.062242	0.063866	0.069057	0.076968	0.086923	0.098390	0.110947	0.124261	0.138072
0.4	0.045199	0.047120	0.051994	0.059118	0.067937	0.078014	0.089002	0.100631	0.112687
0.5	0.031191	0.033237	0.037665	0.043905	0.051510	0.060127	0.069479	0.079351	0.089569
0.6	0.019980	0.021979	0.025828	0.031078	0.037379	0.044460	0.052105	0.060147	0.068454
0.7	0.011378	0.013157	0.016285	0.020428	0.025329	0.030788	0.036650	0.042792	0.049119
0.8	0.005244	0.006624	0.008878	0.011786	0.015178	0.018923	0.022920	0.027089	0.031370
0.9	0.001473	0.002266	0.003482	0.005014	0.006776	0.008705	0.010750	0.012873	0.015045
1.0	0	0	0	0	0	0	0	0	0

Table 1: Threshold cost differential ($\tilde{c}^{LC}$) for different levels of θ and β for Cournot competition (approx. up to six decimal places)

$c\backslash\beta$	0.03	0.06	0.09	0.12	0.15	0.18	0.21	0.24	0.27	0.30
0.03	-	-	-	-	-	-	-	-	-	-
0.03	0.023198	0.054474	0.086593	0.119530	0.153252	0.187716	0.222867	0.258642	0.294963	0.331742
0.06	-	-	-	-	-	-	-	-	0.354298	0.397776
0.06	0.016477	0.047638	0.079593	0.112308	0.145737	0.179826	0.214507	0.249703	0.285324	0.321265
0.09	-	-	-	-	-	0.212650	0.262000	0.308646	0.352944	0.395160
0.09	0.010295	0.041292	0.073033	0.105473	0.138557	0.172217	0.206375	0.240941	0.275813	0.310878
0.12	-	-	0.055560	0.112807	0.166159	0.216318	0.263769	0.308865	0.351867	0.392976
0.12	0.004657	0.035440	0.066916	0.099028	0.131712	0.164890	0.198474	0.232364	0.266449	0.300608
0.15	-	0.008569	0.066402	0.120427	0.171308	0.219508	0.265367	0.309140	0.351020	0.391160
0.15	0	0.030086	0.061242	0.092973	0.125205	0.157850	0.190813	0.223985	0.257250	0.290484
0.18	-	0.020770	0.075328	0.126832	0.175715	0.222293	0.266811	0.309452	0.350363	0.389658
0.18	0	0.025232	0.056014	0. 087310	0.119036	0.151100	0.183398	0.215817	0.248237	0.280531
0.21	-	0.030743	0.082765	0.132254	0.179500	0.224728	0.268110	0.309784	0.349859	0.388422
0.21	0	0.020876	0.051230	0.082038	0.113209	0.144645	0.176239	0.207874	0.239428	0.270774
0.24	-	0.038995	0.089006	0.136858	0.182752	0.226847	0.269269	0.310119	0.349477	0.387408
0.24	0	0.017014	0.046888	0.077155	0.107724	0.138440	0.169343	0.200167	0.230840	0.261237
0.27	-	0.048717	0.094258	0.140766	0.185536	0.228680	0.270290	0.310438	0.349185	0.386577
0.27	0	0.013640	0.042981	0.072659	0.102580	0.132665	0.162718	0.192709	0.222489	0.251938
0.30	0.002780	0.051618	0.098674	0.144096	0.187900	0.230245	0.271170	0.310727	0.348957	o.385893
0.30	0	0.010742	0.039501	0.068544	0.097775	0.127087	0.156370	0.185509	0.214389	0.242894

Table 2: the optimal level of privatization (approx. up to six decimal points) under Cournot (non-shaded row) and Bertrand (shaded row) competition for different cost differential(c) and degree of product substitutability(β) [The blank cells indicates that the rationalizability condition fails to hold for that (c, β). The green cells indicates the optimal degree of privatization under Bertrand competition is greater that that under Cournot competition for those (c, β).]

9.7 Proof of Proposition 5

Proof of Proposition 5(i) To prove our proposition, we simulate $\theta*^{LC}$ for different values of c and β in Table 3. Each row gives the values of $\theta*^{LC}$ for different values of β, when c is fixed at a constant level. We observe inverted-U shaped relation between $\theta*^{LC}$ and β when c is fixed at low level (rows with no color). If c is fixed at a high level, we found U-shaped relationship between the two (green rows). Intermediately, we found a decreasing relationship between them (yellow rows).

By simulation, we found $\bar{c}_{C1} = 0.239$ (approx.) and $\bar{c}_{C2} = 0.415$ (approx.).

Proof of Proposition 5(ii) To prove our proposition, we simulate $\theta*^{LB}$ for different values of c and β in Table 4. Each row gives the values of $\theta*^{LB}$ for different values of β, when c is fixed at a constant level. We observe U-shaped relation between $\theta*^{LB}$ and β when c is fixed at low level (rows with no color). If c is fixed at high enough level, we found a decreasing relationship between them (yellow rows).

By simulation, we found $\bar{c}_B = 0.132$(approx.).

$c\backslash\beta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	-	0.097416	0.113986	0.123058	0.126930	0.126761	0.123390	0.117759	0.111136
0.2	0.246688	0.253375	0.257683	0.260036	0.260251	0.258046	0.253110	0.245230	0.234462
0.3	0.394388	0.388807	0.385893	0.384274	0.382889	0.380833	0.377275	0.371398	0.362394
0.4	0.521339	0.507817	0.499932	0.495799	0.494033	0.493603	0.493698	0.493643	0.492819
0.5	0.631776	0.611972	0.600149	0.594208	0.592739	0.594888	0.600322	0.609318	0.623131
0.6	0.727199	0.701539	0.686113	0.678669	0.677869	0.683222	0.695213	0.715841	0.750313
0.7	0.807238	0.775576	0.756661	0.747922	0.748086	0.757115	0.776552	0.810765	0.871084
0.8	0.869677	0.832034	0.809981	0.800358	0.801906	0.815084	0.824655	0.8918843	0.982073
0.9	0.910450	0.868028	0.843905	0.834249	0.837857	0.855749	0.892023	0.957082	1

Table 3: Optimal level of privatization ($\theta*^{LC}$) for different levels of (c, β) under Cournot competition (approx. up to six decimal place) [Pre-licensing cost differential is fixed along rows. Rows with no color indicates inverted-U shaped relationship between $\theta*^{LC}$ and β; yellow row indicates $\theta*^{LC}$ decreases with β; and green rows indicates U-shaped relationship between $\theta*^{LC}$ and β.]

$c\backslash\beta$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.081651	0.063053	0.049155	0.039692	0.034100	0.031606	0.031382	0.032674	0.034869
0.02	0.192345	0.167972	0.146619	0.128450	0.113442	0.101375	0.091884	0.084524	0.074780
0.03	0.307440	0.274003	0.242894	0.214753	0.189890	0.168338	0.149908	0.134220	0.120452
0.04	0.422740	0.376558	0.333777	0.295081	0.260610	0.230223	0.203597	0.180180	0.158472
0.05	0.532965	0.470902	0.415488	0.366498	0.323269	0.285107	0.251302	0.220929	0.191370
0.06	0.632384	0.552858	0.485107	0.426858	0.376149	0.331493	0.291667	0.255206	0.217962
0.07	0.715600	0.619233	0.540652	0.474758	0.418090	0.368322	0.323663	0.282026	0.237413
0.08	0.778175	0.667873	0.580928	0.509367	0.448373	0.394878	0.346570	0.300700	0.249273
0.09	0.816947	0.697489	0.605285	0.530238	0.466583	0.410729	0.359938	0.310834	0.253478

Table 4: Optimal level of privatization ($\theta*^{LB}$) for different levels of (c, β) under Bertrand competition (approx. up to six decimal place) [Pre-licensing cost differential is fixed along rows. Rows with no color indicates inverted-U shaped relationship between $\theta*^{LB}$ and β; yellow row indicates $\theta*^{LB}$ decreases with β.]

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